CUMULATION, CRASH, COHERENCY: A CRYPTOCURRENCY BUBBLE WAVELET ANALYSIS

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HIGHLIGHTS

- First study of its kind to use intraday data (5-minute resolution).
- Highly significant ($p < 2.2\text{e-}16$) structural change in the relationships between cryptocurrencies towards instability, as indicated by increased wavelet coherence after the Bitcoin peak.
- Robustness of findings demonstrated by extensive sensitivity analysis.
- Visualizations with high time-frequency resolution vividly depicting bubble dynamics.
- A large variety of well-known and lesser-known cryptocurrencies is considered.

ABSTRACT

After its price peaked in 2017, Bitcoin lost almost half of its value in US Dollars within one month, which in turn is likely to have influenced the behavior of market participants — many of who were lay investors. We hypothesize that after the peak, there was a structural change in the relationships between cryptocurrencies towards instability, as indicated by increased interdependence. We utilize wavelet coherence analysis of intraday data (5-minute resolution) to investigate the dynamics between a variety of cryptocurrencies in time-frequency space. Accompanied by visualizations depicting bubble dynamics, our robust and highly-significant quantitative results confirm our hypothesis.

***K*eywords** Cryptocurrencies · Intraday data · Wavelet coherence · Financial bubbles · Market psychology

1 Introduction

Cryptocurrencies quickly became infamous for their propensity to bubble activity and extraordinary return potential. However, from December 2017, they have seen their largest corrections in price in both US Dollars and other fiduciary currencies. Gerlach et al. [1] identified 12/18/2017 as the peak date of the largest bubble in the cryptocurrency market, studying its leading currency, Bitcoin.[1] Fundamental changes in the cryptocurrency market, like the launch of Bitcoin futures contracts at the Chicago Board Options Exchange and the Chicago Mercantile Exchange, align with this peak and the subsequent steep fall of the price of Bitcoin. Bitcoin lost almost half of its value in US Dollars within one month, which in turn is likely to have influenced the behavior of market participants. In the present study, we investigate the development and crash of this bubble, the largest in the cryptocurrency market so far.

We hypothesize that after the Bitcoin price peaked in 2017, there was a structural change in the relationships between cryptocurrencies. Since, in a wide variety of complex systems, measures of dependence have been shown to increase when instability increases – e.g., in human brains with Alzheimer's disease [6], or in equity markets [7] – we assume

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[1]The peak detection algorithm used by Gerlach et al. [1] is based on an extension of the Epsilon Drawdown Method developed by Johansen & Sornette [2, 3] which also has been used in [4] and [5].

that interdependencies of cryptocurrency time series increased, when comparing the weeks before (PRE) and after (POST) the peak of the Bitcoin market price (peak). In other words, we assume that the events in late 2017 led to a fundamental change leading to the instability of the cryptocurrency ecosystem.

Various phenomena of peer influence, describing how market participants are influenced by the decisions of other participants, might contribute to such a structural change. For instance, 'herding behaviour' (for a review, see [8]) describes how individual investors join the crowd in a rush to get in or out of the market. This is often irrational and driven by emotion – greed during rallies, fear during crashes [9]. A concept closely related to herding is momentum in price dynamics, which has strong empirical support from observational studies and the implementation of momentum-based trading strategies [10, 11, 12]. Alternatively, excessive interdependence in market prices might also be caused by delayed reaction to news events [13].

The cryptocurrency market differs from most other financial markets insofar as a high percentage of its participants are lay investors with no or minimal investment history (for a review of characteristics of Bitcoin users, see [14]). These features suggest that phenomena of market psychology might be more pronounced than in other asset markets and that participants might be inclined to act in a more irrational way. This makes the cryptocurrency market a highly interesting field for studying market psychology and the impact of social dynamics in asset markets. In fact, a variety of recent scientific studies have done so. For instance, Glaser et al. [15] found indications that Bitcoin users are limited in their level of professionalism and objectivity, highlighted by their bias towards positive news. Kristoufek [16] observed a bidirectional relationship between web search queries and Bitcoin prices. Garcia et al. [17] detected positive feedback loops between Bitcoin prices, user numbers of the Blockchain network and search queries. They successfully implemented a profitable Bitcoin trading strategy exploiting these social dynamics [18]. Cheah and Fry [19], Gerlach et al. [20], Cobert et al. [21], Bouri et al. [22], and Geuder et al. [23] provided evidence confirming the conclusion that Bitcoin has behaved as a highly speculative asset exhibiting strong bubble activity, whereas Bouri et al. [24] studied the herding behaviour of cryptocurrency market participants, Bouri et al. [25] investigated the relationships between the volatility surprises of leading cryptocurrencies, and Omane-Adjepong & Alagidede [26] examined co-movements and volatility linkages between leading cryptocurrencies. Finally, using an experiment in a real-world online marketplace, Krafft et al. [27] found strong evidence for peer influence among cryptocurreny traders .

In the present investigation, we utilize wavelet coherence analysis as a powerful tool to quantitatively analyze and visualize the dynamics of interdependencies between major cryptocurrencies in time-frequency space. Kristoufek [28] and Phillips and Gorse [29] used this technique to examine potential drivers (e.g., hash rate) of the Bitcoin price, whereas Mensi et al. [30] studied medium- and long-term co-movements between cryptocurrencies' returns series for portfolio risk management, and Bouri et al. [31] examined the safe-haven properties of Bitcoin against the stock market. The wavelet decomposition has several advantages over (short-time) Fourier analysis, including relaxing the assumptions of stationarity, avoiding arbitrary tradeoffs between time and frequency resolutions, entertaining the existence of coherent structure at multiple resolutions as well as allowing efficiency of computation and a sparse representation. To examine associations between variables, we identify coherent time-varying oscillations. Measuring dependence in frequency rather than time allows for phase shifts between signals, which are present between time series of cryptocurrencies.

Our contributions to finance literature are several. As mentioned above, some studies exist which examine the co-movement of cryptocurrencies in time-frequency space. However, all of these utilized data with a temporal resolution of only one data point per day. To the best of our knowledge, not a single study exists in this domain that used intraday data. We address this literature gap by presenting the first study based on higher-resolution data. Thanks to the availability of 5-minute data (288 points per day), we can analyze price dynamics with substantially higher temporal accuracy and closely examine periods of interest around critical events in the market.

Furthermore, we are the first in this domain to particularly investigate, in terms of instability, the impact of the 2017 crash on the structure of the cryptocurrency ecosystem. We therefore investigate interdependencies between pairs of a large variety of both well- and lesser-known cryptocurrencies – considering normalized price and volatility information. Importantly, we not only investigate whether there is higher co-movement after the peak, but, at the same time, higher co-movement during periods of similar magnitude of decline (we identify periods of 72 hours in which the relative difference in the maximum and minimum price of Bitcoin is at least 20%), which would indicate a fundamental change towards instability of the cryptocurrency ecosystem.

Additionally, we provide detailed visualizations with high time-frequency resolution of co-movement dynamics and conduct an extensive sensitivity analysis demonstrating the robustness of our statistical findings.

To establish a reference point for wavelet coherence of Bitcoin, with a currency outside the cryptocurrency market, we examine the dependence between the normalized prices of the Euro and Bitcoin in US Dollars, giving us an indication of whether PRE-POST changes in dependence exclusively occurred within the domain of cryptocurrencies.

Finally, we contribute to the literature by demonstrating the usefulness of price data normalization as a preprocessing step for wavelet coherence analysis in the given domain.

2 Materials and methods

2.1 Data

To examine the development and crash of the biggest cryptocurrency bubble so far, we analyzed data recorded from 11/01/2017 till 05/04/2018 for ten cryptocurrencies traded at the Bittrex exchange (Table 1). At the beginning of the capturing of the data, Bittrex was one of the most important trading platforms in terms of 24 hours trading volume [32].

Table 1: **Characteristics of cryptocurrencies considered in the analysis.**

Rank	Name	Market Cap ($m)	Volume (24h, $m)	Circulating Supply (units, m)
1	Bitcoin	181,619	44,121	18
2	Ethereum	29,599	24,241	109
3	Ripple*	13,565	4,028	43,708
6	Litecoin	5,042	5,633	64
15	Monero	1,571	148	17
17	Ethereum Classic	1,196	2,894	116
18	Dash	1,146	922	9
19	NEO	1,104	1,160	70
28	Zcash	583	706	8
44	OmiseGO*	184	151	140

Rank of cryptocurrencies by market capitalization, name of cryptocurrency, market capitalization (million US Dollars), 24-hour trade volume (million US Dollars), circulating supply (million units), * cannot be mined, all according to coinmarketcap.com (16.02.2020).

2.2 Methodology

2.2.1 Wavelet analysis

When performing a Fourier transform the time series is represented by a sum of harmonic basis functions (sine/cosine components). A wavelet transform instead uses a time-localized basis functions (wavelets) in place of the harmonic basis. A wavelet is characterized by its scale and translation which localize it in frequency and time respectively. A wavelet $\Psi(t)$ fulfills the following two conditions:

$$\begin{aligned} \int_{-\infty}^{+\infty} dt\Psi(t) = 0 \\ \int_{-\infty}^{+\infty} dt|\Psi(t)|^2 = 1 \end{aligned} \tag{1}$$

The first condition, *admissibility*, implies a zero mean. The second condition ensures that the wavelet is localized both in time and frequency space (it has *compact support*). A wide variety of functions give rise to valid wavelets. Here we make use of the Morlet wavelet [33], as it offers good temporal and spectral resolution [34]. The Morlet wavelet is defined as

$$\Psi_0(\eta) = \pi^{-1/4} \mathrm{e}^{i\omega_0\eta} \mathrm{e}^{-\eta^2/2} \tag{2}$$

where η is dimensionless time, and ω_0 is dimensionless frequency [35].

The Morlet wavelet consists of a complex sine wave modulated by a Gaussian envelope. The parameter ω_0 is chosen as 6 as to satisfy the admissibility condition [36]. Given a time series $s(t)$ and a wavelet function $\Psi(\eta)$, the continuous wavelet transform is defined by

$$W_s(u, v) = \int_{-\infty}^{+\infty} dt \frac{1}{\sqrt{u}} \Psi\left(\frac{t - v}{u}\right) s(t) \tag{3}$$

where u and v are scale and translation, which is the location at time t. In the bivariate case, the continuous wavelet transform is then generalized into the cross-wavelet transform. For two series $s(t)$ and $r(t)$ the cross-wavelet transform is given by:

$$W_{rs}(u, v) = W_r(u, v,)W_s^*(u, s) \tag{4}$$

where W^* denotes the complex conjugate of W. As the cross-correlation coefficients are complex numbers they can be interpreted as phase and amplitude in time-frequency (u, v) space. The phase difference[2] between the two series can be

[2]The phase difference informs us about the delays of the oscillations between the two time series [34].

calculated through:

$$\phi_{rs} = \tan^{-1}\left(\frac{\mathcal{I}\left[S(u^{-1}W_{rs}(u,v))\right]}{\mathcal{R}\left[S(u^{-1}W_{rs}(u,v))\right]}\right) \tag{5}$$

where S is a smoothing operator.[3] The cross-wavelet power, $|W_{qs}(u,v)|$, can be interpreted as the covariance between the time series.
The wavelet (magnitude) coherence is defined as a value in the interval $[0,1]$ by the following equation:

$$R^2_{rs}(u,v) = \frac{|S(u^{-1}W_{rs}(u,v))|^2}{S(u^{-1}|W_r(u,v)|^2)S(u^{-1}|W_s(u,v)|^2)} \tag{6}$$

All wavelet coherence plots depicted in the present article were generated using the Cross Wavelet and Wavelet Coherence Toolbox by Grinsted et al. [35] and Matlab 2018a [38].

2.2.2 Volatility

As no commonly agreed upon definition of the calculation of volatility exists, as a straight-forward approach, we used the standard computation of historical volatility. We are aware that other proxies of volatility exist. For instance, Omane-Adjepong and Alagidede [26] used conventional, as well as more complex proxies for volatility when investigating linkages between the volatility of cryptocurrencies. However, they do not provide a statistical analysis of differences between the results of their analysis when employing different volatility proxies.
Let $P \in \mathbb{R}^n$ be the time series of a price of a given currency in US Dollars with k^{th}-entry P_k, and n is the number of data points. We define the volatility to be the moving standard deviation over a period of five hours, using the preceding values (60 data points), i.e., the values used to compute the standard deviation at a given time lie within the range from the previous five hours to the current time step. Since our time steps are in increments of five minutes, we use the preceding 60 data points. Therefore, the volatility $S \in \mathbb{R}^{n-60}$ with k^{th}-entry S_k at time $k+60$ (where the unit of time is 5 minutes) is defined to be

$$S_k = \sqrt{\frac{1}{59}\sum_{i=k}^{k+60}|\mu_i - P_i|^2}, \tag{7}$$

where μ_k is the moving average that is defined to be

$$\mu_k = \frac{1}{60}\sum_{i=k}^{k+60} P_i, \tag{8}$$

for $k = 1, 2 \ldots n-60$.

2.2.3 Normalization

When computing the wavelet coherence between the prices of different currencies in US Dollars, we normalize the prices of the time series by dividing the moving average by the moving standard deviation. If $P \in \mathbb{R}^n$ is the time series of a price in US Dollars of a particular cryptocurrency, we define the normalized price $\hat{P} \in \mathbb{R}^{n-60}$ with k^{th}-entry corresponding to time $k+60$ as follows:

$$\hat{P}_k = \frac{P_k - \mu_k}{S_k}, \tag{9}$$

for $k = 1, 2, \ldots n-60$.

2.2.4 Statistical analysis

For the statistical PRE-POST comparison of wavelet coherence we identified all periods of 72 hours duration in which the price of Bitcoin decreased by at least 20% against the US Dollar (drops). After square-root transformation and Fisher's r-to-z transformation, coherence data was averaged into bands of short- (<1 hour), medium- (1–24 hours), and

[3] The smoothing is done using a weighted running average (or convolution) in both the time and scale directions. For further details on the smoothing operator used, see [37].

long-term (>24 hours) duration.[4] As we did not want to include the biased data inside the Cone of Influence[5] (COI) in our computations, we restricted our analyses to period bands below 128 hours. Then, data was partitioned into matrices containing the previously identified drops. These matrices were then horizontally concatenated for further analysis.
We defined the PRE-POST change in wavelet coherence between the Bitcoin and Ethereum price series as the primary outcome measure. To test differences between PRE and POST, we utilized a robust linear model with PRE-POST as the independent variable, and transformed coherence values as the dependent variable. The averaged short-, medium-, and long-term period bands were added as covariates. To account for the time series nature of the data, a robust covariance estimator, also known as 'sandwich estimator', was used to compute the test statistics ('sandwich' R package [40]). As heteroskedasticity and autocorrelation consistent covariance matrix estimator, the package's implementation of the Newey-West method [41] was utilized with default settings.
To test the robustness of our model, we carried out a wide range of sensitivity analyses for the primary outcome measure. As for the robust linear model described above, we varied the input using no normalization at all, and normalization with distinct duration (5 hours instead of 3). We also tested the inclusion of data inside the COI, i.e., using data of all period bands. Furthermore, we used alternative robust, heteroskedasticity and autocorrelation consistent covariance estimators (kernel-based using different kernel weights following [42] and [43]) to assess the impact of the covariance estimator on the results. Finally, we modeled the data utilizing a mixed model (dependent variable, transformed coherence values; independent variable, PRE-POST; independent random intercepts for individual frequency bands and time steps; lmerTest R package [44] with default settings).
All statistical analyses were run in R (version 3.5.1) [45].

3 Results and discussion

3.1 Price

Table 2 reports the statistical PRE-POST analyses. All combinations of currencies – including Bitcoin-Ethereum (primary outcome) – yielded highly significant results, even after Bonferroni adjustment of the significance level for multiple comparisons (α = 0.006). Hence, our hypothesis was confirmed. Furthermore, results of all variants of the sensitivity analysis were highly significant, demonstrating the robustness of our finding.
Fig 1 depicts a wavelet coherence plot between Bitcoin and Ethereum and indicates the drops that were statistically investigated. For all of these 14 drops, Fig. 2 shows distinct wavelet coherence plots with a higher level of detail.

Table 2: **PRE-POST comparisons (normalized price).**

	t	p
Bitcoin-Ethereum	17.8970	< 2.2e-16**
Bitcoin-Dash	24.2700	< 2.2e-16**
Bitcoin-Ethereum Classic	17.1057	< 2.2e-16**
Bitcoin-Litecoin	20.5900	< 2.2e-16**
Bitcoin-NEO	15.0418	< 2.2e-16**
Bitcoin-OmiseGO	17.7234	< 2.2e-16**
Bitcoin-Zcash	15.7300	< 2.2e-16**
Bitcoin-Monero	21.2210	< 2.2e-16**
Bitcoin-Ripple	13.1811	< 2.2e-16**

A linear model with robust covariance estimate ('sandwich estimator' [40], based on the Newey-West method [41]) was used to compute the test statistics. t-statistics and corresponding p-values (two-tailed probabilities). * significant at $\alpha = 0.05$, ** significant at $\alpha = 0.006$ (9-fold Bonferroni correction).

Visually comparing the short-term dynamics of drops before (Fig 2 (a) - (c)) and after the peak (Fig 2 (d) - (n)), not only can we find an increase in the area of significant coherence (in line with the statistics reported), but also that these are longer-lasting. These differences are particularly pronounced in the period bands above 30 minutes. Relationships of shorter duration are more sparse and erratic in general. Considering medium-term relationships before the peak, significant coherences predominantly arose along the eight-hour period band. After the peak, dependence between

[4] Please note that these band definitions are not universal, but specific to the present investigation.

[5] As the wavelet transform is not completely localized in time, edge artifacts occur. For this reason a COI is introduced in which the wavelet transform is influenced by the edge effect [35]. The COI is defined as the e-folding time, an interval where a quantity increases by a factor of e^{-2}. The quantity here is chosen to be the autocorrelation of wavelet power at each scale [39].

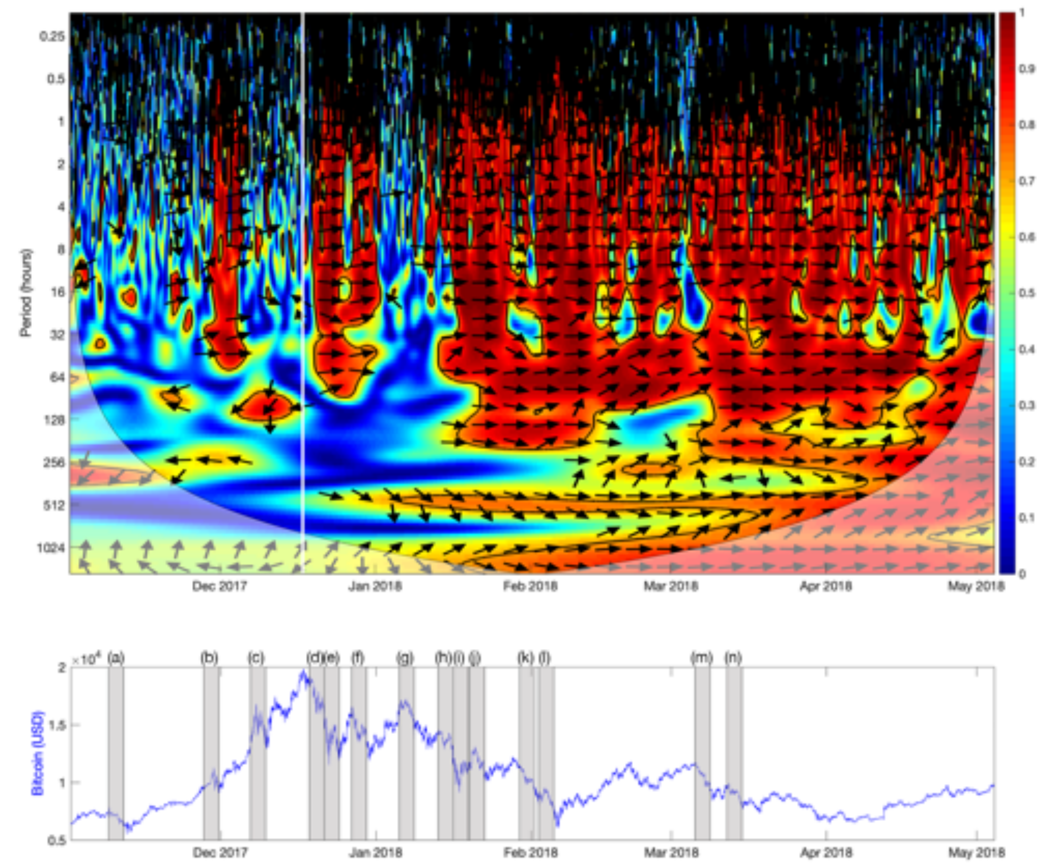


Figure 1: **Wavelet coherence between Bitcoin and Ethereum (normalized price).** Gray overlays (a)-(n) indicate periods of 72-hour during which the price of Bitcoin decreased by at least 20% (drops). Arrows indicate areas where the wavelet coherence exceeds 0.5. Their directions show the phase lag (unit circle) of the altcoin with respect to the Bitcoin. For instance, a quarter-cycle phase lag of the altcoin, which suggests that the Bitcoin is leading the other currency, is indicated by a downward-pointing arrow. The phase angle can also be interpreted as a time lag for a given wavelength (the time lag depends on the cycle duration). However, since there is no unique interpretation of the results, a lead of 90 degrees (in-phase) might as well be interpreted as a lag of 90 degrees (anti-phase). Arrows pointing horizontally to the left or right indicate maximal negative or positive correlations respectively. Areas of significant coherence are enclosed within black line contours, whereas a white line (December 2017) indicates the peak of the Bitcoin market price. Several drops coincide with prominent historical events concerning Bitcoin: (a) 8 November 2017: Developers cancel splitting of Bitcoin [46], (f) 28 December 2017: South Korea announces strong measures to regulate trading of cryptocurrencies [47], (h)-(i) 13 January 2018: Announcement that 80% of Bitcoin has been mined [48], (k)-(l) 30 January 2018: Facebook bans advertisements promoting, cryptocurrencies [49], (m) 7 March 2018: The US Securities and Exchange Commission says it is necessary for crypto trading platforms to register [50], (n) 14 March 2018: Google bans advertisements promoting cryptocurrencies [51].

Bitcoin and Ethereum increased and became more spread out over different periods.[6] For the drops before the peak, the phase arrows do not tend to point in any particular direction. In contrast, after the peak, the arrows almost exclusively point to the right, thus indicating a positive correlation.[7]

Fig 3 shows the wavelet coherence between Bitcoin and all the altcoins. Due to the high temporal resolution of the data and the depiction of significant[8] areas of coherence – indicated by black lines – the upper part of the diagram is obscured. When plotting without the significance borders (plots not shown here), we discovered that there are erratic, highly coherent regions in all subplots. Persistent medium-term significant dependencies start to occur in mid-December 2017 to early January 2018 (depending on subplot) for all combinations of currencies. These dynamics occur across all medium-term period bands roughly equally and they are rarely interrupted. With very few exceptions, the phase arrows point to the right, indicating a positive correlation. With the exception of ZCash, the plots display highly coherent regions along the 512-hour period band, starting in January 2018. The arrows mostly point upwards, hence indicating a lead by Bitcoin.

The effect of price normalization is demonstrated in Fig 5. As this figure shows, not normalizing the price might lead to biased results.[9] Looking at Fig 6(a), which depicts the wavelet coherence of volatility between Bitcoin and Ethereum, one might assume that part of the coherence depicted in the non-normalized price plot can be explained by volatility.

In order to have a reference point of wavelet coherence, i.e. between Bitcoin and a currency outside the cryptocurrency domain, we investigated the dependence between the normalized prices of the Euro and Bitcoin in US-Dollars (Fig 4). Importantly, there is no visible PRE-POST difference in subplot (b). This finding was confirmed by exploratory statistical testing ($t = 0.2005$, $p = 0.8411$). These results indicate that the observed PRE-POST changes in dependence exclusively occurred within the domain of cryptocurrencies.

3.2 Volatility

For the PRE-POST test statistics of all combinations of cryptocurrencies investigated, see Table 3. The corresponding wavelet coherence plots are depicted in Fig 6. All statistical tests were highly significant.

Due to the high resolution of the data, the significance borders impair the visibility of the short-term coherences. However, we can examine the high-frequency dynamics between Bitcoin and Ethereum in Fig 7.

The wavelet plots of Bitcoin and altcoins, as depicted in Fig 6, are very similar. Regarding the short-term and medium-term dynamics, significant areas mainly occur after the peak. For the long term, this is only true for particular combinations, like Bitcoin-Ethereum or Bitcoin-Ripple. In contrast to the price dynamics (see Fig 3), some plots display persistent long-term coherences that are almost unchanged after the peak.

Interestingly, for certain currencies (e.g., Bitcoin-NEO) the phase arrows tend to point upwards along the high period bands (256-1024 hours), thus indicating a lag of the volatility of Bitcoin.

3.3 Comparison to previous studies

Comparing our results to previous studies, we have to restrict our analysis to the long-term frequency band (>24 hours duration), since all of the existing investigations have used daily data points. In line with Mensi et al. [30], Omane-Adjepong and Alagidede [26], Bouri et al. [22], and Phillips and Gorse [29] we find evidence of significant co-movements between Bitcoin and other cryptocurrencies. Like Omane-Adjepong and Alagidede [26] we find non-homogeneous directions of linkages between currency pairs, and strong interdependencies, especially at the low frequency bands.

Bouri et al. [25] have shown that volatility surprises do not necessarily originate from the largest cryptocurrency. Interestingly, we also find that Bitcoin is not the only leading cryptocurrency – for instance, we find NEO taking the lead along the high period bands (256-1024 hours) of volatility lasting for many weeks.

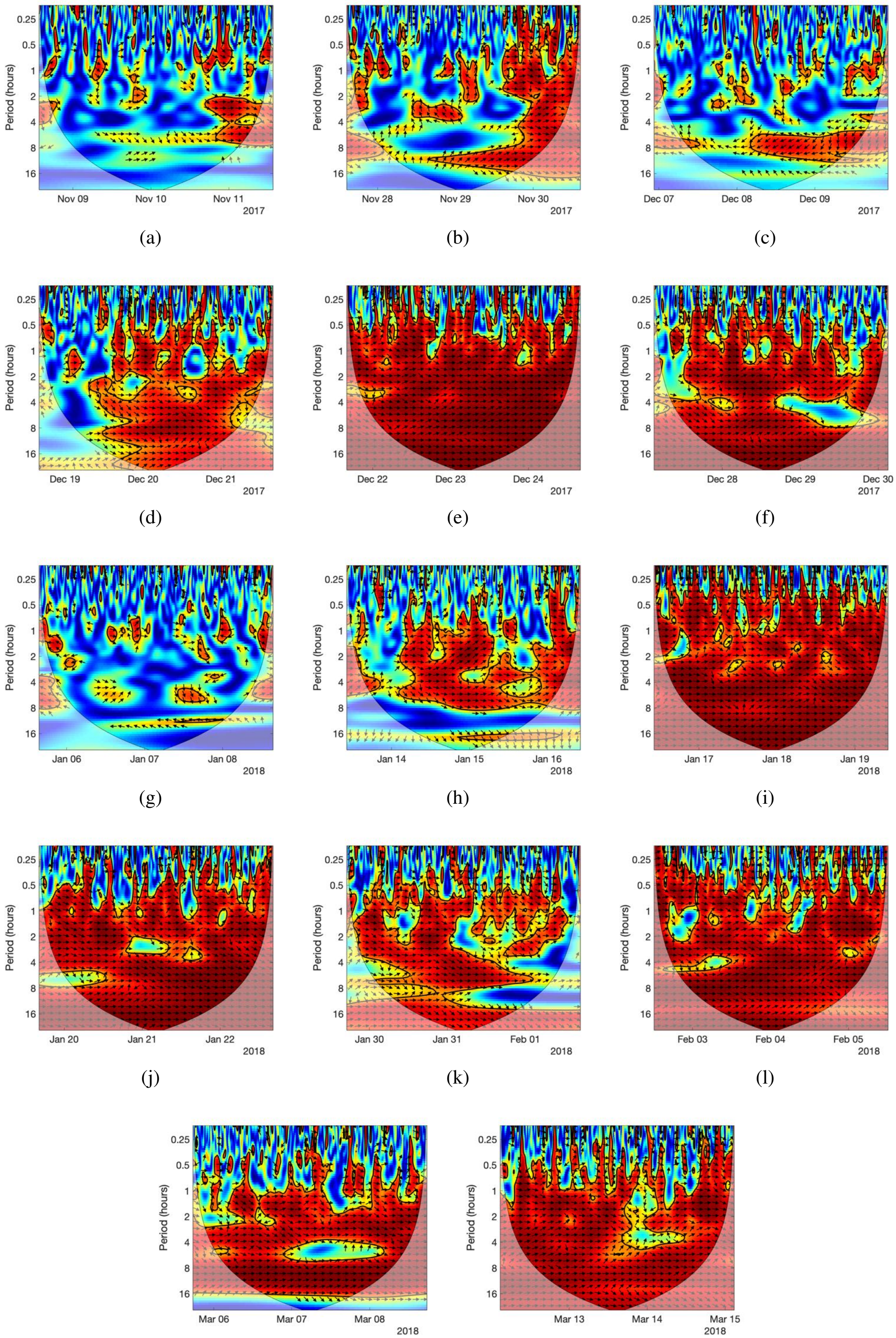
Mensi et al. [30] report significant areas of coherence between major cryptocurrencies in their wavelet coherence plots,

[6] Interestingly, subplot (g) shows very little area of significant coherence, and therefore looks more similar to a subplot of a drop before than a drop after the peak. Inspecting Fig 3, this might be not completely surprising, since the Bitcoin market seems to have started rallying again in early January 2018. However, after another massive decline, many investors may have lost their hope in a quick recovery of Bitcoin.

[7] Once more, subplot (g) constitutes an exception with the phase arrows pointing in multiple directions.

[8] Significance analysis is done by comparing the results to a null hypothesis that the processes examined are not coherent. The distribution under the null hypothesis is estimated by using a Monte Carlo simulation. This distribution is taken to be red noise, or in other words, an autoregressive process of order one. The number of surrogate data sets in the significance calculation here equals 300, following previous work [35].

[9] For instance, the wavelet coherence plot using the non-normalized prices (a) shows a significant positive long-term correlation (512-hour period band) between Bitcoin and Ethereum for the entire observed duration. The plot depicting the normalized price (b) also shows significant areas along the same period band, however, these only appear after the peak.



(a) (b) (c)

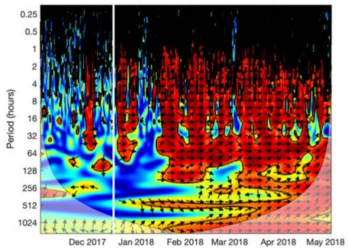
(d) (e) (f)

(g) (h) (i)

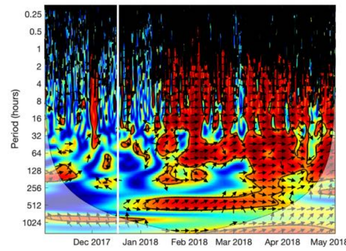
(j) (k) (l)

(m) (n)

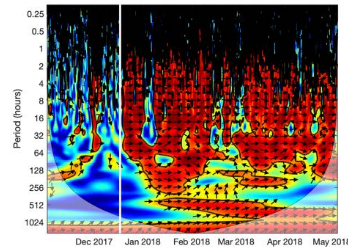
Figure 2: **Wavelet coherence between Bitcoin and Ethereum of drops (normalized price).** Drops are defined as 72 hours periods during which the price of Bitcoin decreased by at least 20%. The subplots (a) - (n) are presented in chronological order (see Fig 1).



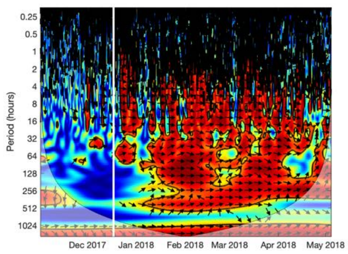
(a)



(b)



(c)



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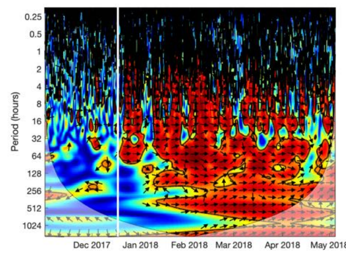
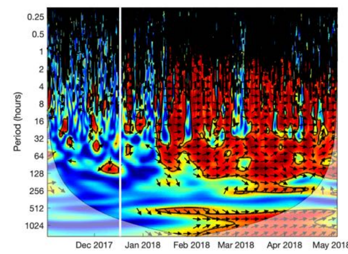
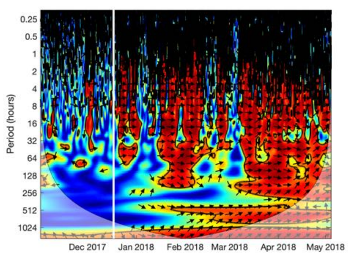


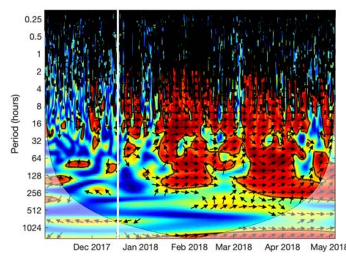
(e)



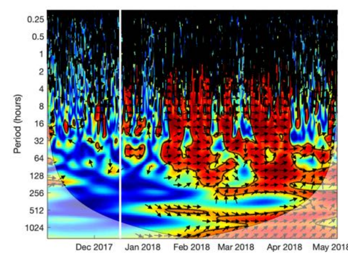
(f)



(g)

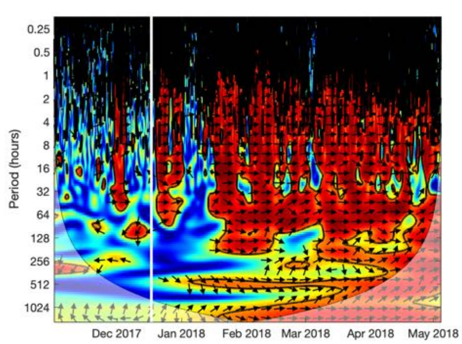


(h)

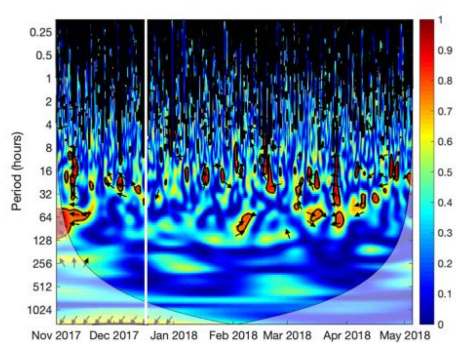


(i)

Figure 3: **Wavelet coherence between Bitcoin and other cryptocurrencies (normalized price).** (a): Bitcoin-Ethereum, (b): Bitcoin-Ripple, (c): Bitcoin-Litecoin, (d): Bitcoin-Monero, (e): Bitcoin-DASH, (f): Bitcoin-NEO, (g): Bitcoin-Ethereum Classic, (h): Bitcoin-ZCash, (i): Bitcoin-OmiseGO.

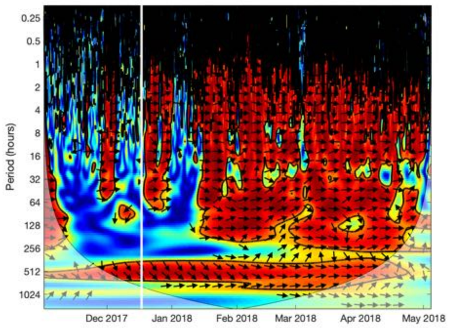


(a)

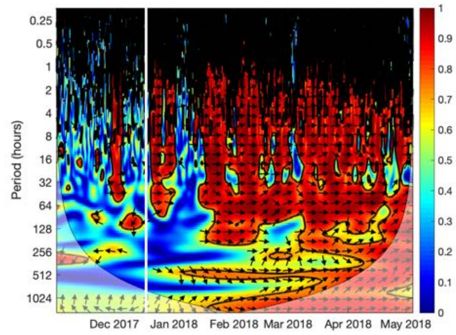


(b)

Figure 4: **Wavelet coherence between Bitcoin and Ethereum vs. Bitcoin and Euro.** (a): Bitcoin-Ethereum, (b): Bitcoin-Euro (both in US Dollars). Because of the missing trading data on weekends and holidays, the normalization method (as reported in Section 2.2.3) had to be adapted when normalizing traditional foreign exchange data. We normalized by using the average and standard deviation of the most recent trading day. In order to avoid artefacts, the price of Bitcoin was normalized the same way for this plot.



(a)



(b)

Figure 5: **Wavelet coherence between Bitcoin and Ethereum (non-normalized vs. normalized price).** (a): non-normalized price, (b): normalized price.

Table 3: **PRE-POST comparisons (volatility).**

	t	p
Bitcoin-Ethereum	17.9960	$<$ 2.2e-16**
Bitcoin-Dash	24.6310	$<$ 2.2e-16**
Bitcoin-Ethereum Classic	10.2590	$<$ 2.2e-16**
Bitcoin-Litecoin	12.9800	$<$ 2.2e-16**
Bitcoin-NEO	13.0370	$<$ 2.2e-16**
Bitcoin-OmiseGO	12.8340	$<$ 2.2e-16**
Bitcoin-Zcash	15.9300	$<$ 2.2e-16**
Bitcoin-Monero	17.9170	$<$ 2.2e-16**
Bitcoin-Ripple	9.4933	$<$ 2.2e-16**

A linear model with robust covariance estimate ('sandwich estimator' [40], based on the Newey-West method [41]) was used to compute the test statistics. t-statistics and corresponding p-values (two-tailed probabilities). * significant at $\alpha = 0.05$, ** significant at $\alpha = 0.006$ (9-fold Bonferroni correction).

but to a much smaller extent than we do. This difference in magnitude can be explained by the fact that their time window of analysis ends long before ours, and the authors rightly exclude large parts of their data in 2018, since it lies inside the COI, thereby omitting areas in time frequency space with particularly pronounced co-movement.

Phillips and Gorse [29], who extended the work by Kristoufek [28], examined various potential drivers of the Bitcoin price in US Dollars as well as the relationships between three cryptocurrency price series. The periods investigated end long before December 2017, thereby not including the arguably most interesting period in the history of cryptocurrencies so far, the peaking of the Bitcoin price and subsequent major crash. However, like Phillips and Gorse [29] we find a particularly strong interdependence between Bitcoin and Litecoin, which might be caused by their technical similarity (Litecoin is a fork of the Bitcoin Core client, differing primarily by having a faster transaction confirmation). Omane-Adjepong and Alagidede [26] also report this finding.

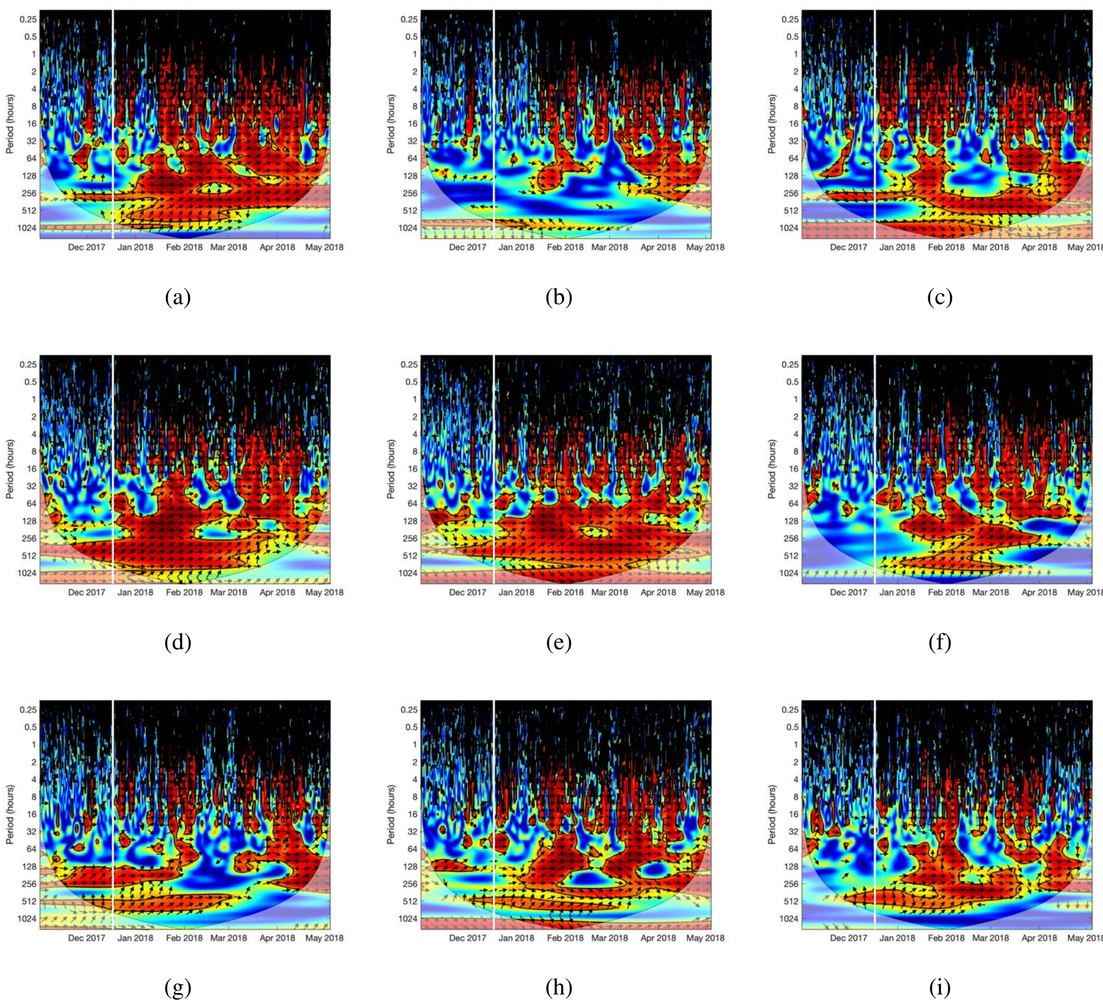


Figure 6: **Plots of wavelet coherence between Bitcoin and other cryptocurrencies (volatility).** (a): Bitcoin-Ethereum, (b): Bitcoin-Ripple, (c): Bitcoin-Litecoin, (d): Bitcoin-Monero, (e): Bitcoin-DASH, (f): Bitcoin-NEO, (g): Bitcoin-Ethereum Classic, (h): Bitcoin-ZCash, (i): Bitcoin-OmiseGO.

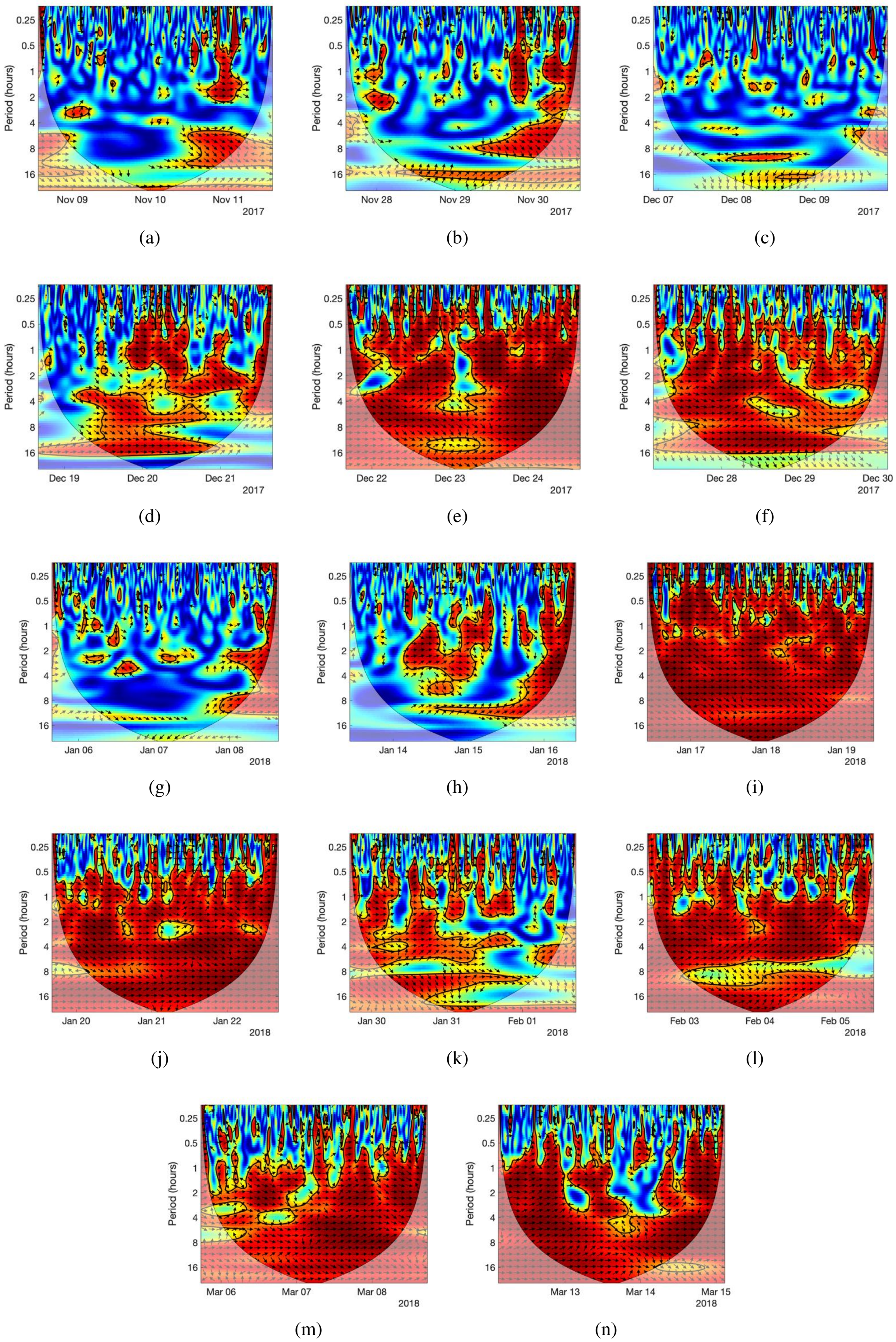


Figure 7: **Wavelet coherence between Bitcoin and Ethereum of drops (volatility).** Drops are defined as 72-hour periods within which the price of Bitcoin decreased by at least 20%. The subplots (a) - (n) are presented in chronological order (see Fig 1).

4 Conclusions

Using wavelet coherence analysis we showed that after the peak of the Bitcoin price in December 2017, there was a structural change in the relationships between cryptocurrencies towards instability, as indicated by increased interdependence. This result adds to a growing body of literature showing that the dependence between the parts of a complex system increases, when instability increases. In asset markets in general – and the cryptocurrency market with its high percentage of lay investors in particular – this phenomenon is thought to be driven by market psychology.

Details on dependencies and phase lags between cryptocurrencies are of interest to those seeking risk diversification opportunities for portfolio management, as well as to those using forecasting methods for achieving excess returns from market inefficiencies. Our findings indicate the potential to predict normalized price as well as volatility dynamics, which might be examined by scholars in future studies, or exploited by market participants. Using wavelet coherence matrices based on intraday data as input for machine learning methods (such as convolutional neural networks), opportunities for algorithmic trading might exist. Furthermore, our results highlight the advantages of analyzing cryptocurrencies in time-frequency space and using data with a high temporal resolution. Since characteristics of co-movement and directionality differ between time scales, distinct opportunities and risks are indicated for portfolio management. Finally, the fundamental structural change in the co-movement patterns between cryptocurrencies after the peak (significant increase in dependence during periods of similar magnitude of decline) highlights the importance of using dynamic instead of static models for trading as well as risk diversification.

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